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Technique for the Accurate Positioning of Objects in an Ice-Covered Ocean

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Abstract

A standard procedure in the underwater acoustics community is to determine the location of an array of hydrophones with acoustic techniques. Unfortunately, it is very hard to measure the accuracy of such techniques since there is nothing to which to compare the results. This report describes a way of suspending an array under the ice of the Arctic Ocean in such a way that the hydrophone locations are known accurately. Weights are suspended from the ice by three lines, and the array is stretched between the weights. This system was tried out near Alert, Nunavut, Canada in the spring of 2000, and this report discusses the various strengths and weaknesses of the support system. Accuracies of about 10 cm were obtained. A number of suggestions are made for simplifying the installation.

Résumé

En acoustique sous-marine, la détermination de l'emplacement d'un réseau d'hydrophones suit une procédure normalisée utilisant des techniques acoustiques Il est cependant très difficile de déterminer la précision d'une technique de ce genre car on ne dispose d'aucun point de comparaison pour les résultats. Le rapport décrit une méthode qui consiste à suspendre un réseau d'hydrophones sous la glace de l'océan Arctique, de telle manière que l'emplacement des hydrophones est connu avec précision. Des poids sont suspendus au bout de trois câbles fixés dans la glace et le réseau est déployé entre les poids Ce système a été mis à l'essai près d'Alert (Nunavut) au Canada, au printemps 2000, et le rapport fait état des points forts et des points faibles du système de câbles de soutien. L'exactitude des valeurs est d'environ 10 cm. Un certain nombre de propositions ont été faites pour simplifier l'installation.

Executive summary

INTRODUCTION:
The standard technique for determining the locations of underwater hydrophones is to detonate small explosions or to use a c.w. noisemaker and infer the locations from the signals received by the hydrophones. This is known as 'array element location' (AEL). Unfortunately, it is very difficult to test the accuracy of these techniques since there is nothing to compare them to. 'Ground truthing' is very difficult.

With this as motivation we have developed a method of accurately placing an array of hydrophones under the ice of the Arctic Ocean. These accurately known locations can then be used as a comparison with the positions that are determined by the standard acoustic methods.

SUMMARY OF DISCUSSION:
A weight supported from the ice surface by three lines is very stable in position Moreover, its location can be calculated from a knowledge of the ice holes positions, the three line lengths and the thickness of the ice at the three support locations. Our method involves stretching the array between two such accurately placed weights.

The report discusses the mathematics of determining the location of the weights and the tensions in the support lines. It takes account of the corrections that are necessitated by the stretch of the cable, and it discusses the errors due to water currents A spreadsheet was developed so that these calculations would all be instantly available in the field

In the spring of 2000 this concept was tried out as part of an expedition to the Arctic Ocean north of CFS Alert, Nunavut Underwater lines were strung with the help of an ROV (Phantom), and an installation was made. The errors in the calculated positions were measured by drilling a hole above the upper weight and lowering a measuring tape (with a 10-lb ball) down to the weight. The ROV was able to tell us when the ball was at the right height. This gave us the actual depth of the weight and its horizontal distance from the calculated position This error was about 30 cm. However, if the use of the measuring tape is considered as part of the measurement procedure, the position error is estimated to be about 5 to 10 cm.

The report goes on to discuss the strengths and the weaknesses of the technique, and it makes a number of suggestions for simplifying future installations.

SIGNIFICANCE OF RESULTS:
Underwater acoustic arrays are of great value to the Canadian Forces, and it is of importance to have a good knowledge of the shape of the array, i.e., the locations of all the hydrophones. Since these locations are usually measured acoustically, it is of importance to know the accuracy of these acoustic methods. This suspension technique will allow for 'ground truthing' of the acoustic methods.

FUTURE PLANS:

This report does not include the analysis of the acoustic aspects of the experiment We intend to do this work in the near future. We also intend to do more array element location work as part of the 'Rapidly Deployable Systems' Technology Demonstrator. Specifically, we hope to create a rapidly deployable array that has its own AEL capability In addition, the positioning method is sufficiently general that other uses for it might arise at any time

Verrall, R. I and Heard, Garry J 2000. Technique for the Accurate Under-Ice Positioning of Objects in an Ice-Covered Ocean, DREA TM 2000-103 Defence Research Establishment Atlantic.

Sommaire

INTRODUCTION:

La technique normalisée pour déterminer l'emplacement des hydrophones sous-marins consiste à provoquer de petites explosions ou à utiliser un générateur de bruit, puis à déduire l'emplacement de l'hydrophone grâce aux signaux émis par ce dernier. Il s'agit là de l'emplacement des éléments du réseau, appelés AEL, de l'anglais « Array Element Location ». Il est cependant très difficile de déterminer la précision de telles techniques car on ne dispose d'aucun point de comparaison pour les résultats. La vérification au sol est très difficile.

Pour régler ce problème, nous avons mis au point une méthode permettant de mettre en place un réseau d'hydrophones d'emplacement connu sous la glace de l'océan Arctique. Ces emplacements connus avec exactitude peuvent être utilisés à titre de comparaison pour les positions qui sont déterminées par les méthodes acoustiques normalisées.

RÉSUMÉ DE L'ANALYSE

La position d'un poids soutenu par trois câbles fixés dans la glace est très stable. De plus, l'emplacement du poids peut être calculé si l'on connaît la position des trous dans la glace, la longueur des câbles et l'épaisseur de la glace aux trois emplacements de soutien. Notre méthode comprend l'étalement du réseau entre deux poids dont l'emplacement est connu avec exactitude. Le rapport présente les aspects mathématiques de la détermination de l'emplacement des poids et des tensions dans les câbles. Il tient compte des corrections requises à cause de l'étirement des câbles, et il traite des erreurs imputables aux courants marins. Une feuille de calcul a été mise au point et permettra que ces données soient consultées sur le terrain Au printemps 2000, le projet a été mis à l'essai dans le cadre d'une expédition dans l'océan Arctique au nord de la SFC Alert (Nunavut). Les câbles sous-marins ont été placés à l'aide d'un engin télécommandé (Phantom) et une installation a été aménagée. Les erreurs dans les positions calculées ont été mesurées en forant un trou au-dessus du poids supérieur et en descendant un ruban à mesurer (au bout duquel était attachée une boule de 10 lb) vers le poids. L'engin télécommandé nous a permis de savoir que la boule se trouvait à la bonne profondeur. Nous avons ainsi obtenu la profondeur réelle du poids et sa distance horizontale par rapport à la position calculée L'erreur est d'environ 30 cm. Toutefois, si l'utilisation du ruban à mesurer est considérée comme une partie de la procédure de mesure, l'erreur relative à la position est estimée à environ 5 à 10 cm.

Le rapport traite également des points forts et des points faibles de la technique, et fait un certain nombre de suggestions pour simplifier les installations à venir.

IMPORTANCE DES RÉSULTATS

Les réseaux d'acoustique sous-marine sont très importants pour les Forces canadiennes, et il faut bien connaître la forme du réseau, c'est-à-dire l'emplacement de tous les hydrophones Comme ces emplacements sont habituellement mesurés au moyen de méthodes acoustiques, il faut connaître avec précision les méthodes acoustiques. Cette technique de suspension permettra de compenser l'absence de « vérification au sol » des méthodes acoustiques.

PLANS À VENIR

Le rapport ne comprend pas l'analyse des aspects acoustiques de l'expérience. Nous avons l'intention de faire ces travaux dans un avenir rapproché. Nous avons également l'intention d'effectuer d'autres travaux de détermination de l'emplacement des éléments de réseaux avec le démonstrateur de la technologie des « systèmes à déploiement rapide ». Plus particulièrement, nous souhaitons créer un réseau à déploiement rapide qui peut déterminer de manière autonome l'emplacement des éléments d'un réseau En outre, la méthode de positionnement est générale et pourrait avoir d'autres utilisations.

Verrall, R. I. and Heard, Garry J. 2000 Technique for the Accurate Under-Ice Positioning of Objects in an Ice-Covered Ocean [Enter report no.] Defence Research Establishment Atlantic.

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1.0 Introduction:

The solid, unmoving ice of the Arctic Ocean can provide a very stable platform for suspending equipment at accurately known positions. In the spring of 2000 we used the stable ice north of Alert, Nunavut, to support an array of hydrophones in precise locations for the purpose of testing acoustic techniques that are described below. In this report we describe the suspension method used, and we discuss its strengths and weaknesses. We give the mathematics for determining the location of the supported objects and for estimating the errors that water currents might introduce. We also discuss ways of simplifying the installation and improving the technique.

Our use of the ice for accurate underwater positioning was motivated by a procedure that is in fairly common usage in southern waters, namely the use of acoustics to determine the location of hydrophones in an underwater array. A standard 'array element localization' (AEL) technique is to implode light bulbs or to place continuous wave (c.w.) sources in the vicinity of the hydrophone array and to measure the time intervals or phase differences between the various hydrophones. Several mathematical techniques have been developed to determine the hydrophone locations that would give a 'best fit' to the data[1,2,3,4]. However, the accuracy of these methods is hard to determine since there is generally no way of checking the hydrophones' locations with a more trustworthy method.

In the spring of 2000 we carried out a test in which we deployed an array of hydrophones at accurate locations under Arctic ice and then used the standard acoustic methods to measure where they were. A comparison of these acoustically determined locations with the 'known' locations of the hydrophones will give us a measure of the accuracy that can be ascribed to the acoustic techniques.

This paper describes the technique that we used for the accurate emplacement of the hydrophones This method is, of course, not limited to positioning hydrophones; it is a general approach that could be used to place anything accurately under the ice. An analysis of the acoustic experiments will be presented elsewhere.

2.0 Technique for the Accurate Positioning of an Array:

Although the solid ice of the Arctic Ocean provides a stable platform from which to suspend an array, some thought must be given to the details of this suspension. A weight simply suspended by a single rope, for example, would move with the current. One needs to constrain the object in all three dimensions.

Our method of positioning an object accurately is to support it by three ropes, as shown in Figure 1 The position of a weight suspended this way is very stable, a large lateral force would be required to move it. And, of course, the heavier the weight and the more spread out the support points, the more solid the structure. One can think of the mechanism as being an upside-down tripod

Although this three-point suspension is very simple in conception, there are a number of practical considerations that are important to recognize First of all, it is necessary to be able to calculate the weight's location from a knowledge of the positions of the ice holes, the thickness of the ice at each hole and the lengths of the three ropes Also, for accurate positioning, it is necessary to have a technique for leading the rope vertically through the ice and a way of knowing accurately the location of the bottom of this hole, i e , the point where the rope turns from the vertical toward the suspended weight. Moreover, it is necessary to consider the elasticity of the support lines and the effect of water current.

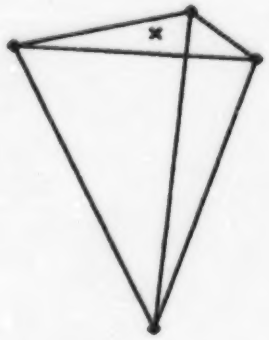

FIGURE 1 Weight supported by three lines fixed at the ice surface

The precise positioning of a single heavy weight locates only one end of the array of hydrophones. One method of supporting the rest of the array would be simply to hang it from the weight. This has the disadvantage mentioned before that the array would be free to swing in the current And even if there were no current, this technique has the weakness that the array orientation would not be controllable, its only orientation would be vertical.

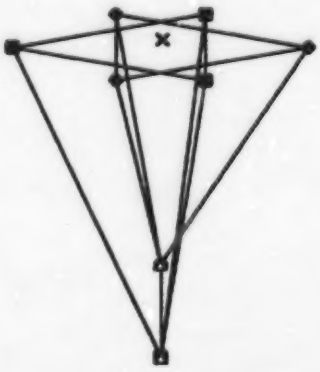

FIGURE 2 Two weights being supported, each with its own upside-down tripod The hydrophone array is slung between the two weights

A possible way of controlling the bottom end might be to sling another heavy weight from the surface and attach the bottom of the hydrophone array to it This arrangement (known as

the Cat's Cradle) is shown in Figure 2. One difficulty with this is that there are now four lines (three support ropes plus the array) supporting the bottom weight, and one of them is bound to be slack. This would mean that either the array would not be straight or that the bottom weight would not be where we think it is

A solution to this problem is to put an extensible member (some form of spring) in series with the array. This allows the array to be in tension without forcing its total length to be fixed. The spacing between the two ends of the array is adjusted so that the tension in the array is large enough to keep it straight, but not so large that the lower weight will be lifted from its intended position. This was the solution that we adopted. Problems with this method and ideas for its improvement are presented in Section 7.0.

3.0 Analysis

3.1 General

Equations were developed to analyze the geometry of the structure, the forces within it, the effect of the elasticity of the support ropes and the effects of a water current. Then, to analyze the many 'what if' questions pertaining to the design and execution of the experiment, a spreadsheet was developed to do the calculations quickly. The advantage of a spreadsheet over other types of programming is that the input parameters are easily viewed and easily changed. Likewise, the output is instantly available, and this output includes a visual display of what the underwater structure looks like from an arbitrary point of view (e g., Figure 2). The spreadsheet, which was placed on a laptop computer, was indispensable in the field

The following section describes the analysis of the geometry, the tensions in the ropes and the deformation in the structure caused by a current. It also gives an indication of how this is put into operation within the spreadsheet.

3.2 Location of the Suspended Body

We assume that each of the underwater objects is being suspended by three ropes from three different locations on the ice (as shown in Figure 2). For the initial estimate of the object's location, we assume that there is no current. Also, it is very reasonable to assume that the ropes are neutrally buoyant in water. With these assumptions, the ropes extend in a straight line from the ice to the weight.

The ice holes are assumed to be located by a range and a bearing from an 'origin' on the ice surface This corresponds to our practice of surveying locations with a Geodimeter transit The actual bearing that corresponds to 0° on the transit will, of course, depend on the installation at hand. From the range and bearing, the coordinates (x, y) relative to the local coordinate system are easily calculated.

The finite thickness of the ice adds a complication to the calculation since the ropes do not go directly from the top of the hole to the weight; they go vertically to the bottom of the ice hole before going off at an angle to the weight More precisely, they go to the bottom of the pipe with which the hole is lined. Consequently, the calculation of the position of the weight requires knowledge of the ice thickness at each of the six holes. Note that for this purpose the 'ice thickness' is defined as being the distance from the water surface to the bottom of the pipe - not the distance from the top of the ice to the bottom of the ice The water surface is the most stable horizontal plane available, and, whenever possible, measurements are carried out relative to it. The 'z' coordinate in our calculation is taken to be zero at the water surface and positive in the downward direction.

We now calculate the location of the weight in terms of the rope lengths and the ice thicknesses From Figure 3 we can see that the length of support rope 1 is given by:

$$L_1 = t_1 + \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-t_1)^2}\,, \qquad (1)$$

where t_1 is the ice thickness as defined above

Rearranging and squaring, we obtain:

$$x^2 - 2x_1x + y^2 - 2y_1y + z^2 - 2t_1z = (L_1 - t_1)^2 - x_1^2 - y_1^2 - t_1^2 = RHS_1. \qquad (2)$$

And, similarly for the other two ropes holding up this weight,

$$x^2 - 2x_2x + y^2 - 2y_2y + z^2 - 2t_2z = (L_2 - t_2)^2 - x_2^2 - y_2^2 - t_2^2 = RHS_2, \qquad (3)$$
$$x^2 - 2x_3x + y^2 - 2y_3y + z^2 - 2t_3z = (L_3 - t_3)^2 - x_3^2 - y_3^2 - t_3^2 = RHS_3, \qquad (4)$$

where (x_2, y_2, t_2) and (x_3, y_3, t_3) are the coordinates of the bottoms of the other two ice holes.

We wish to solve these three equations for x, y and z. Subtracting equation (3) from equation (2) we obtain

$$2(x_2 - x_1)x + 2(y_2 - y_1)y + 2(t_2 - t_1)z = RHS_1 - RHS_2. \quad (5)$$

Similarly, by subtracting equation (4) from equation (2), we get

$$2(x_3 - x_1)x + 2(x_3 - y_1)y + 2(t_3 - t_1)z = RHS_1 - RHS_3. \quad (6)$$

These equations are linear in x, y and z. A third equation could be obtained by subtracting equation (4) from equation (3), but it would not be independent of equations (5) and (6) since the same equation could also be obtained by subtracting equation (5) from equation (6). Thus, we do not have three independent linear equations for an easy solution. One of the necessary three equations must be non-linear, and this complicates the problem.

The situation can be saved, however, by noting that the coefficient of 'z' in equations (5) and (6) is very small In fact, if the ice thickness 't' were everywhere the same, the coefficient of 'z' would be zero, and we would have two linear equations in two unknowns, 'x' and 'y'. In that case, we would solve (5) and (6) for 'x' and 'y' and substitute back into equation (2), say, for 'z'.

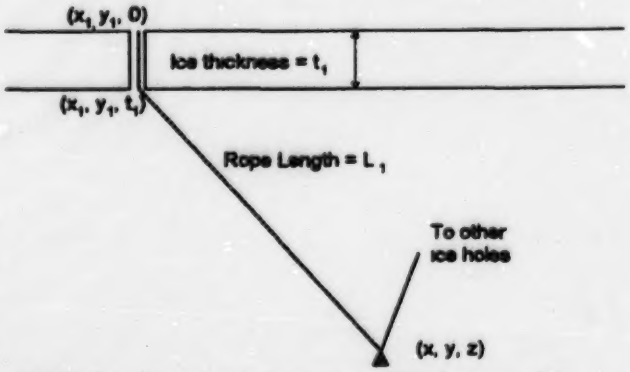

FIGURE 3 This sketch shows the salient features of the geometry

Although the coefficient of 'z' is not zero, it is small, and we can use a modification of the above procedure. We estimate (guess) a value for 'z' and then solve (5) and (6) for 'x' and 'y'. We back-substitute into (2) to obtain a new value of 'z' and iterate this procedure until the values of 'x', 'y' and 'z' stop changing. Two iterations after the initial calculation are enough for our geometries, and this procedure was entered into the spreadsheet

3.3 Isometric View of the Structure

Once the positions of the two weights have been calculated, the location of all parts of the structure is known. A useful tool at this point is to be able to look at it from various points of view. One sheet of the spreadsheet is devoted to producing an isometric view of the two weights and their support cables from any direction desired. The user enters the bearing and elevation of the direction from which he is looking, and the spreadsheet calculates

the (x,y) position on the projection plane of all the relevant parts of the structure It plots these points on a spreadsheet 'chart' and draws lines between the appropriate points

The mathematics of isometric projections is sufficiently straightforward that only the results are reproduced here Let 'B' be the bearing (clockwise from north) of the direction to the observer and 'E' be the elevation angle of the observer. Let (x,y,z) be the position in space that is being projected onto the projection plane, and let (u,v) be the corresponding x and y coordinates on the projection plane. In this calculation 'v' is positive upward but 'z' is positive downward Then:

$$u = -x \cos(B) + y \sin(B) \quad (7)$$

$$v = -(x \sin(B) + y \cos(B)) \sin(E) - z \cos(E). \quad (8)$$

Figures 1 and 2 are examples of isometric projections generated by the spreadsheet.

3.4 Calculation of the Tensions in the Supporting Ropes

It is important that we know the tensions in the support ropes since the stability of the structure depends on all of the tensions being adequately large. Reasonably large tensions are required to nullify the effect of water current and the weight of the rope, and, therefore, we must ensure that nothing we do causes the tension in any rope to become unacceptably smal!

Once we know that the tensions are high enough to ensure the stability of the structure, we make further use of these tensions to recalculate the positions of the two weights. A knowledge of the elasticity of the support rope (discussed further in Section 5 1), the tension and the (original) length of the rope enables us to calculate the rope's increase in length With the new lengths we can then recalculate the positions of the weights (as discussed in Section 3 2).

In order to calculate the tensions in the supporting ropes, we first need to know the total force pulling at the junction of the three ropes. This will include the gravitational pull of the weight as well as any other force that is being applied to the weight In the case described here, another pull is caused by the tension in the array strung between the two weights. This tension is determined from the spring constant of the array and the separation of the two weights that are supporting the two ends of the array. The direction of this force is determined by the orientation of the array, i.e., by the displacement of one weight relative to the other In practice, the calculation of this force and its direction was done

automatically by the spreadsheet. The drag force of the current was ignored since we will only do the experiment when the current is very light. Its drag force will be only a small fraction of the other forces in the structure.

The other important factor in determining the tensions is the geometry of the support structure: in other words, 'what are the angles at which the ropes leave the weight'? Since we know the coordinates of both ends of each rope, we have these directions.

The approach taken to calculate the tension in Line 1, say, is to write down the equation that says that the moment of all the forces on the weight about a particular line is equal to zero. The best line for this calculation is the one running from the bottom of the ice at Hole 2 to the bottom of the ice at Hole 3. This line is chosen so that the (unknown) tensions in lines 2 and 3 do not contribute to the equation; their moments about the line are zero.

In the subsequent equations, the bold-faced type represents vectors. Let $\mathbf{r}_1$ be the vector from the origin to the bottom of Hole 1, (i.e., $\mathbf{r}_1 = (x_1,y_1,z_1)$). The position vectors $\mathbf{r}_2$ and $\mathbf{r}_3$ are similarly defined. (The origin can be taken anywhere since only vector differences are used) Let $\mathbf{r}_w$ be the position vector from the origin to the weight supported by lines 1, 2 and 3 Let $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$. This is the position vector from Hole 2 to Hole 1. Similarly, let $\mathbf{r}_{1w} = \mathbf{r}_1 - \mathbf{r}_w$, etc.

At the junction of the three ropes let $\mathbf{F}_w$ be the (known) force due to the pull of gravity plus the pull of the array. The force of gravity is, of course, vertically downward, but, as discussed above, the pull of the array is directed toward the other weight and is not necessarily vertical However, since the coordinates of both weights are known, the components of this direction (and, therefore, the components of the force) are straightforward to calculate.

Let $\mathbf{F}_1$ be the force on the weight due to the tension in line 1. Its magnitude (and that of $\mathbf{F}_2$ and $\mathbf{F}_3$) are the objects of the exercise.

The tension F_1 is calculated, as described above, by writing down the moment of the total force $\mathbf{F}_1 + \mathbf{F}_w$ around the line running between the points defined by $\mathbf{r}_2$ and $\mathbf{r}_3$ and setting this moment equal to zero. (Again, the moments of $\mathbf{F}_2$ and $\mathbf{F}_3$ about this line are zero, and so they are ignored in what follows.) The desired moment can be calculated by taking the cross product of $\mathbf{F}_1 + \mathbf{F}_w$ with $\mathbf{r}_{2w}$ (or with $\mathbf{r}_{3w}$) and then taking the component of the resulting vector in the direction of $\mathbf{r}_{23}$. It is this component that must be zero In other

words,

$$(\mathbf{F}_1 + \mathbf{F}_w) \times \mathbf{r}_{2w} \cdot \mathbf{r}_{23} = 0. \tag{9}$$

The dot and the cross products can be interchanged, giving

$$(\mathbf{F}_1 + \mathbf{F}_w) \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{23}) = 0. \tag{10}$$

Now $\mathbf{r}_{23} = \mathbf{r}_{2w} - \mathbf{r}_{3w}$, and since $\mathbf{r}_{2w} \times \mathbf{r}_{2w} = 0$, we have that the above cross product is

$$\mathbf{r}_{2w} \times \mathbf{r}_{23} = \mathbf{r}_{2w} \times (\mathbf{r}_{2w} - \mathbf{r}_{3w}) = -\mathbf{r}_{2w} \times \mathbf{r}_{3w} \tag{11}$$

This identity reduces the number of vectors that need to be calculated in the spreadsheet since no vectors of the form $\mathbf{r}_{23}$ are needed Equation (9) is now:

$$(\mathbf{F}_1 + \mathbf{F}_w) \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{3w}) = 0 \text{ or } \mathbf{F}_1 \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{3w}) = -\mathbf{F}_w \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{3w}). \tag{12}$$

Although the force $\mathbf{F}_1$ has an unknown magnitude, it has a known direction, which is along the vector heading from the weight to hole 1. Thus the force can be written as the magnitude F_1 times a unit vector in the $\mathbf{r}_{1w}$ direction.

$$\mathbf{F}_1 = F_1 \mathbf{r}_{1w} / r_{1w} \tag{13}$$

When this is placed in equation (12) above, the product $\mathbf{r}_{1w} \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{3w})$ occurs. This (scalar) is known as the box product of the three vectors. It is the volume of the parallelepiped formed by the three vectors, and it will be denoted here as $[\mathbf{r}_{1w}, \mathbf{r}_{2w}, \mathbf{r}_{3w}]$. Equation (12) reduces to

$$F_1 = -\mathbf{F}_w \cdot (\mathbf{r}_{2w} \times \mathbf{r}_{3w}) \mathbf{r}_{1w} / [\mathbf{r}_{1w}, \mathbf{r}_{2w}, \mathbf{r}_{3w}] \tag{14}$$

This can be computed from the information contained in the spreadsheet. The forces in the other two lines are calculated by cyclic permutation of the subscripts in equation (14). Note that the box product does not change in value as the subscripts are cycled.

A similar analysis of the other weight will give the tensions F_4, F_5 and F_6.

3.5 Distortions Caused by Current Drag

We ignored the current drag when calculating the tensions in the support ropes since we do not need high accuracy in this calculation; we simply need to know that the tensions are large enough to ensure stability of the structure. However, water flowing past the support

ropes will cause them to 'belly out' downstream, and this will reduce the distance from the ice hole to the weight. Since we desire quite high accuracy in the positioning of the weights, this effect must be examined. Such a study will help us decide on the magnitude of the weights, the diameter of the ropes and the maximum current that we can live with.

In order to calculate the shape of the curved rope in an analytic fashion (i.e., without resorting to numerical integrations) two simplifying assumptions were made, both of which are quite reasonable. The rope was assumed to be weightless in water, and the tangential drag of the current on the rope was assumed to be zero. The result of these two assumptions is that the tension in the rope is constant over its whole length and that the shape of the rope is a catenary. The current pushing on the whole structure will slightly increase the tensions in the upstream cables and lower them in the downstream cables. This effect, however, has been ignored in the following calculations.

The catenary shape is derived as follows Figure 4 shows the rope being pushed by the water current We will take a coordinate system with the (horizontal) x-axis in the same direction as the current and the z-axis upward Although we are taking the z-axis upward for reasons of familiarity, we will later interpret upward on the paper as being downward in the water

The rope element of length dS shows the forces on this element Note that there is no force of gravity and no drag component parallel to the rope. The drag component normal to the rope is assumed to be proportional to the square of the normal velocity component (as shown in Figure 4). The other terms in the equation are:

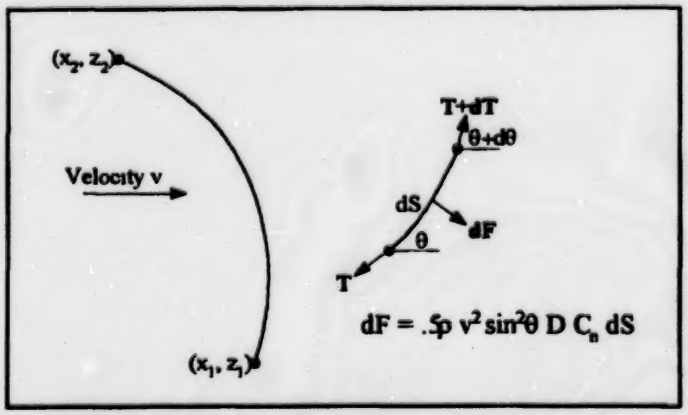

FIGURE 4 Schematic showing the rope pushed sideways by the water current On the right is a rope element showing the forces on it

ρ = density of water
D = diameter of the cable
C_n = the 'normal' drag coefficient.
θ = the angle the rope makes with the horizontal.
S = the arc length of the cable.

Since the sum of the forces parallel to the rope element equals zero, we have that $dT = 0$, which implies that the tension, T, is everywhere constant.

Similarly, setting the sum of the forces perpendicular to the rope element equal to zero, we have:

$$T\, d\theta = dF = 0.5\, \rho\, v^2 \sin^2(\theta)\, D\, C_n\, dS = TA \sin^2(\theta)\, dS \tag{15}$$

or,

$$\csc^2(\theta)\, d\theta = A\, dS, \tag{16}$$

where 'A' lumps together the terms independent of θ. Anticipating the form of the solution, we use z as our independent variable. Taking the derivative of the expression for the reciprocal slope

$$dx/dz = \cot(\theta), \text{ we obtain } \quad d^2x/dz^2\, dz = -\csc^2\theta\, d\theta \tag{17}$$

and

$$dS = \sqrt{dx^2 + dz^2} = \sqrt{1 + \left(\frac{dx}{dz}\right)^2}\, dz \tag{18}$$

When equations (17) and (18) are substituted into equation (16), we obtain.

$$\frac{d^2x}{dz^2} = -A \sqrt{1 + \left(\frac{dx}{dz}\right)^2}\,. \tag{19}$$

We try a solution of the form $x\text{-}x_0 = -f \cosh(g(z\text{-}z_0))$, where 'f', 'g', x_0 and z_0 are constants Substituting this into equation (19), we find that it is a solution if $g = 1/f = A$. In other words, the solution is:

$$x\text{-}x_0 = -(1/A) \cosh(A(z\text{-}z_0)), \tag{20}$$

where $A = 0.5\, \rho\, v^2\, D\, C_n /T$, and the constants x_0 and z_0 depend on the details of the boundary conditions This is the equation of a catenary.

The arc length between (x_1, z_1) and (x_2, z_2) is of particular importance to us It is obtained by evaluating and integrating equation (18) between z_1 and z_2. The integration yields the arc length

$$S = 1/A\, (\sinh A(z_2\text{-}z_0) - \sinh A(z_1\text{-}z_0)) \tag{21}$$

We now have the basic catenary equations, and we must decide on a strategy for using them on our system in which three cables are interrelated by being tied together at their bottom points. In reality, each rope will belly out by an amount that depends on (among other things) the length of the cable and the angle the cable makes with the current The result of the cable deformations is that the distance from the weight to each of the holes will decrease. The main effect of all three lengths being shortened is a lifting of the weight and a somewhat smaller shifting of the weight sideways.

A determination of the new weight location involves the solution of three non-linear equations, and these are quite complicated to solve. However, neither the drag coefficient nor the dependence of the drag on the angle to the current is known accurately enough to merit such a complicated approach. Moreover, even if the input parameters and the differential equation were known precisely, the accuracy associated with an exact solution is really not necessary for our purposes. We require only a reasonable estimate of the effect of the current so that we can be assured that the position errors are smaller than some allowable amount. In practice, for example, this may involve waiting until the current goes below some particular value

The approach we have taken is to use the coordinates of the rope ends as calculated in the absence of a current and determine the rope length that would be necessary to join these points when there is a current. If this calculated increase in rope length is small we can infer that (at least to first order) the actual shortening of the distance between hole and weight will be roughly the same as this calculated increase.

Let us look at the problem of computing the arc length of a rope joining two given points in space when the rope is subjected to a current flowing in an arbitrary (but given) direction. First of all we note that the catenary lies in a plane. (This would not be true if gravitational forces were important.) This plane is determined by the two end points of the rope and by the direction of the current.

Let $\mathbf{r}_{12}$ be the vector between the top and bottom ends of the rope

Let the velocity of the current $\mathbf{v} = v(\mathbf{i}\sin(B) + \mathbf{j}\cos(B)) = v\mathbf{i}'$. (22)

Here 'B' (for 'bearing') is the direction of the current's direction (measured clockwise from the local north, which is the y-axis in our coordinate system) . The vector $\mathbf{i}'$ is a unit vector in this direction The two vectors, $\mathbf{r}_{12}$ and $\mathbf{i}'$ define the plane of the catenary.

We would like to generate a unit vector **k'** perpendicular to **i'** such that **i'** and **k'** also define the plane The vector $\mathbf{r_{12}} \times \mathbf{i'}$ is perpendicular to the plane, and $\mathbf{i'} \times (\mathbf{r_{12}} \times \mathbf{i'})$, being perpendicular to this, is back in the plane. It is also perpendicular to **i'**. This, then, is in the direction of the unit vector we want. To evaluate the double cross product more easily in the spreadsheet we use the identity,

$$\mathbf{i'} \times (\mathbf{r_{12}} \times \mathbf{i'}) = \mathbf{r_{12}} - (\mathbf{r_{12}}\ \mathbf{i'})\mathbf{i'} \tag{23}$$

To make this into the unit vector **k'** we normalize it by dividing by its length.

We can now calculate the components of $\mathbf{r_1}$ and $\mathbf{r_2}$ in the primed coordinate system For example, $x_1' = \mathbf{r_1}\ \mathbf{i'}$, and $z_1' = \mathbf{r_1}\ \mathbf{k'}$. The coordinates x_2' and z_2' are evaluated similarly The information that we extract from the new values of the coordinates is the differences in x and z. That is, $\Delta x = x_2' - x_1'$ and $\Delta z = z_2' - z_1'$

In the new plane we can write the equation of the catenary (from equation (20)) as

$$x'' = -(1/A)\cosh(Az'') \tag{24}$$

Note that we have moved the origin to the as-yet-unknown location where x_0 and z_0 in equation (20) are zero However, the changes Δx and Δz are still the same. Thus we have

$$x_2'' - x_1'' = \Delta x = -(1/A)(\cosh(Az_2'') - \cosh(Az_1'')) \tag{25}$$

We also know that $z_2'' = z_1'' + \Delta z$, which can be used in equation (25) to yield

$$\Delta x = -(1/A)(\cosh(A(z_1'' + \Delta z)) - \cosh(Az_1'')). \tag{26}$$

The only unknown in this equation is z_1''. For conciseness we write

$Az_1'' = h$, $A\Delta z = \Delta h$, and $A\Delta x = \Delta g$.

With these substitutions, equation (26) becomes

$$\Delta g = -(\cosh(h + \Delta h) - \cosh(h)). \tag{27}$$

We solve for 'h' by expressing the cosh function in terms of exponentials.

$$2\Delta g = -(\exp(h + \Delta h) + \exp(-h - \Delta h) - \exp(h) - \exp(-h)). \tag{28}$$

Multiplying through by exp(h) and rearranging, we obtain a quadratic in exp(h)

$$a \exp(2h) + b \exp(h) + c = 0, \text{ where} \qquad (29)$$
$$a = \exp(\Delta h) - 1, \quad b = 2\Delta g \text{ and } c = \exp(-\Delta h) - 1.$$

We solve this quadratic for exp(h) and then take its logarithm to get h. For this logarithm to be real, at least one of the roots of the quadratic must be positive. If we assume that Δh is positive, then an examination of the terms in the quadratic equation shows that 'a' is positive, 'c' is negative and 'b' may be of either sign. This implies that the quadratic has exactly one positive root. Thus, we are assured that there is one and only one physically realistic solution.

Once we have 'h', we can easily calculate z_1'' and z_2'', and from equation (24) we can solve for x_1'' and x_2''. We now have the coordinates of the top and bottom points in the double-primed coordinate system. The length of the rope can then be calculated from equation (21), with $z_0 = 0$. This gives

$$S = 1/A \, (\sinh(Az_2'') - \sinh(Az_1'')). \qquad (30)$$

A comparison of this length with the straight-line distance is the primary object of the exercise The spreadsheet computes these differences for all six lines. It then calculates a new approximate position of each weight by assuming that the distance from hole to weight has been decreased by this difference.

Another interesting parameter is the maximum amount by which the rope departs from the straight line. In other words, how much does it 'belly out'? This maximum departure is calculated by writing down the perpendicular distance from an arbitrary point on the catenary to the straight line between (x_1'', z_1'') and (x_2'', z_2'') and then maximizing this distance.

From plane analytical geometry we know that the distance of a point (x", z") from the line is

$$D = (x'' - x_1'')(z_2'' - z_1'')/s - (z'' - z_1'')(x_2'' - x_1'')/s, \qquad (31)$$

where 's' is the distance between the two points (x_1'', z_1'') and (x_2'', z_2''). For brevity we rewrite this as

$$D = \alpha(x'' - x_1'') - \beta(z'' - z_1'') \qquad (32)$$

For x" we use equation (24), which says that: $x'' = -(1/A)\cosh(Az'')$. Substituting this into equation (32) and maximizing the distance 'D' with respect to z", we obtain

$$z''_{max} = -(1/A)\sinh(\beta/\alpha) \qquad (33)$$

The maximum distance D_{max} is found by substituting this value of z''_{max} and its associated value of x''_{max} (from equation (24)) into equation (32). This, then, is a measure of how much the catenary deviates from a straight line. This value for each of the six ropes is calculated by the spreadsheet

4.0 Results of a Planning Study

The spreadsheet that incorporated these calculations was invaluable in the Arctic to tell us the location of the weights and the orientation of the array. However, it was also extremely useful for planning purposes. It helped us choose the location of the holes, the length of the ropes and the size of the weights to be suspended

Once we had decided that two weights could be profitably used to control the location of the array, our initial thought was that the two sets of ropes could be deployed down the same set of three holes. After all, surveying and drilling three ice holes and stringing three under-ice lines is less work than doing all that for six holes.

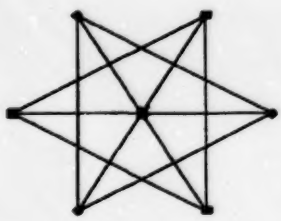

FIGURE 5 Top view of support structure This configuration minimizes the interference between the lines

However, a little thought and visualization soon convinced us that the two sets of ropes would interfere with each other, especially when the array was being manoeuvred from a vertical orientation towards the horizontal. To minimize this interference the two sets should be separated from each other as much as possible For example, when each set of ropes forms a triangle in plan view, the interference is minimized when one triad of ropes is rotated 60 degrees relative to the other. The resulting plan view looks like a Star of David (Figure 5)

A disadvantage of this strong symmetry showed up when we started pulling (in simulation) the bottom weight to one side in order to tilt the array off the vertical. As one supporting rope was shortened, the spreadsheet indicated that the tension in the other two rapidly went to zero This problem was mitigated by moving the hole for the pull rope farther away from the

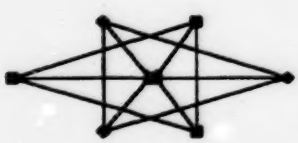

FIGURE 6 Here each triangular support has one hole that is placed farther out than the others. Its rope is the one used to pull the weight to the side

origin. Likewise, the rope providing the opposing force on the upper weight also needed to be moved farther away. Figure 6 shows the improved geometry. As working distances we took one ice hole in each triplet of holes to be 65m away from the origin and the other two to be 40 m from the origin. The array is initially placed vertically below the origin It can be pulled into a horizontal position by pulling up on the outlier rope to the bottom weight and by pulling up slightly the other two ropes to this weight. At no time in the process did the calculated forces ever drop by more than a quarter of their original values.

5.0 Design Details: Choice of Materials, etc.

5.1 Support Ropes

The ropes that suspend the weights from the ice surface have two desired properties that conflict with each other. On one hand, the rope needs to be resistant to stretching since any elongation will produce an error in the calculated position of the weight. (To some extent this error can be removed by calculating the extension of the rope and then recalculating the positions of the weights However, approximations were made in the calculations of the tensions, and so we wish the correction to be as small as possible.) On the other hand the rope should have a small diameter so that it is not unduly affected by water currents, but a small-diameter rope will stretch more than one that is larger. In addition, the rope should preferably be neutrally buoyant, strong, flexible and easy to handle

Light kevlar cord, which is both inelastic and very strong, seems to have a mix of the most desirable characteristics. We decided on a 3/32-in.-diameter kevlar line whose force-strain curve is shown in Figure 7. This plot says that a 100-m length subjected to a 20-lb. pull would be stretched by about 0.16 m or 16 cm. This curve is used in the calculations described in Section 3 4. For the sake of comparison, this cord is about 40 times as 'stiff' as a nylon rope of the same diameter and 1/13th as stiff as a steel wire of the same diameter.

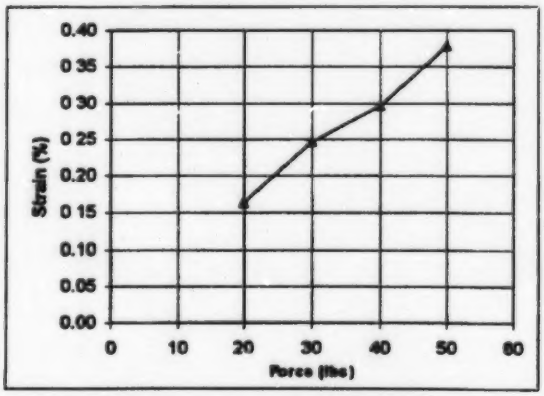

FIGURE 7. Force-strain curve for 3/32-in kevlar woven cord

Before going to the Arctic, we cut six 120-m lengths of this cord, and we measured and marked it every 5 metres.

5.2 Support Spring for the Array

In our suspension technique, the hydrophone array is tensioned between the two weights with a force large enough to keep it straight but not so large that the tension in one of the lower support ropes decreases to a dangerously low value. Moreover, we wanted the array/spring system to be capable of rather large elongations since the two weights may be moved relative to one another, and we did not want to be forced to control their relative separation to the nearest few inches. On the other hand, we did not wish the spring to have a particularly long initial length. This means that the spring material must be capable of undergoing large strains An easily obtainable material that has these properties is surgical rubber hose Its stress-strain relationship is not particularly linear, but the deviation from linearity did not appreciably affect our calculations of the rope tensions (Our definition of strain is the usual engineering one: strain is the elongation divided by the original length)

A latex surgical rubber tubing that seemed quite adequate for our purpose has a diameter of 7/16 inches and an inside diameter of 1/4 inch It has the force-strain characteristic shown in Figure 8

This plot shows that a strain of 2 requires a force of about 17 lbs., and a strain of 3 requires a force of about 25 lbs., and this was a suitable range of forces considering that the weights were approximately 40 to 60 lb. Preliminary tests indicated that the tubing behaved well at low temperatures It did not become stiff as most rubbers and plastics do. However, see Section 7.3 for comments on the behaviour of this material during the field trial.

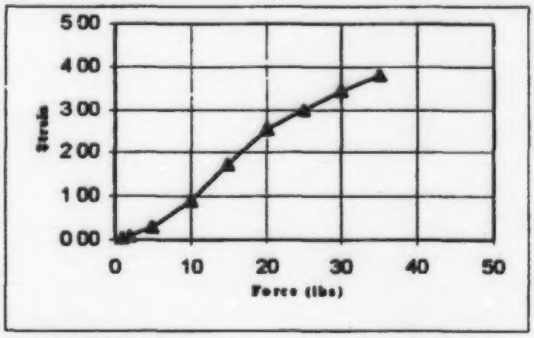

FIGURE 8 Force-strain relationship for 7/16-in surgical rubber hose

5.3 Pipe for Lining the Ice Hole

Because we were after position accuracies of approximately 10 cm we needed to know the precise location of the bottom of each ice hole. The top of the hole could be surveyed

quite accurately. The difficulty was knowing the location of the bottom relative to the top. To get the accuracy we wanted, we had to make the hole as plumb as possible, we had to prevent the bottom of the hole from becoming distorted, and we had to prevent the fine (3/32") cord from cutting into the soft ice at the bottom of the hole. We decided that the best way to accomplish all this was to line the hole with a two-inch ABS plastic pipe.

For a short while we considered using steel pipe. We were worried that if the pipe extended down into the water below the hole it might bend to the side if it were plastic, and this would produce an error in the position of the bottom of the pipe Steel, on the other hand, would be stiff enough to remain straight. We finally ruled out the steel pipe primarily because it is so hard to cut and thread in the field. ABS plastic, on the other hand, is very easy to cut to the right length or to extend should the ice be thicker than expected. The plastic has the additional advantages of being cheaper, lighter and less likely to cut the light kevlar rope.

To keep the kevlar line from cutting into the bottom of the pipe we glued an eighth-bend (22 5°) elbow to each pipe bottom. Its slight curvature was adequate to keep the line from cutting the plastic.

The concern about the pipe bending to the side at the bottom of the hole was handled in the following way First, we measured the thickness of the ice carefully so that we knew how much pipe to put down the hole. We let it protrude only an inch or so. Secondly, when the hole was being drilled with the hot-water drill, we were careful to make the hole at the bottom of the ice just large enough for the pipe. As a result, the pipe was supported laterally right at the bottom of the ice hole, and not a foot or so above the bottom. The hole was made the right size by running the hot water down the pipe itself in the latter stages of the drilling process When the pipe fell through, we stopped the water flow.

We made the top of the hole substantially larger so that there was enough clearance that we could make the pipe plumb - as indicated by a spirit level. Once the pipe was vertical, we packed snow around it to hold it in place. We then left it for a day or so until it froze solidly into place, after which we resurveyed the top of the pipe.

6.0 Installation of the Array - Results and Observations

6.1 Location and Lay-Out of Camp

We established the ice camp on the Arctic Ocean at 82° 32' 21",N, 62° 36' 26" W, just 6 km north-west of Alert, Nunavut (See Figure 9)

The camp was north-east of William Island in the centre of a large expanse of first-year ice that ranged in thickness from 1.4 to 1.7 m The water depth was 47 6 m at the centre of the camp. For the most part, the surface of the ice was quite smooth, although a few places showed small sharp ridges that were formed when the ice was quite thin. Except for 'our' pan of first-year ice, most of the ice in the area was several years old, the rolling surface being ablated and smoothed by the melt of several summers

The layout of the camp and the positioning of the holes for the array are shown in Figure 10 After we drilled and lined the six holes for the array support cables, we cut a large (4 x 7 ft) hole through the ice for the remotely operated vehicle (ROV) Phantom, which we used to string all the under-ice lines necessary for the array deployment We put the Phantom hole well away from the underwater ropes because Phantom had other jobs to do, and we did not want it to become tangled in the ropes

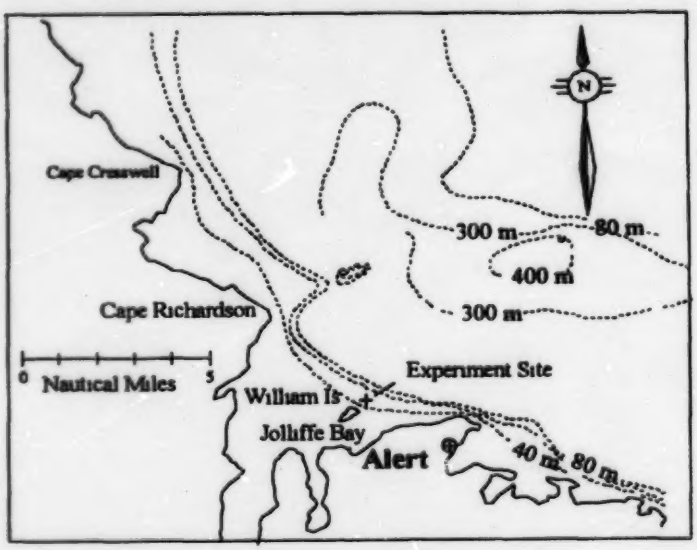

FIGURE 9 Map of the Alert area showing Jolliffe Bay and William Island

6.2 Stringing the Lines

To string each underwater rope, we first melted out the ice in the appropriate frozen-in ABS pipe (Figure 11). Then, down a nearby 4-inch hole we lowered a flashing underwater light to help guide the Phantom ROV to the vicinity. (We make it flash since a steady light at a distance cannot be distinguished from the many dots of light shining through the ice.) Once Phantom arrived at the hole, the light was pulled up, and the kevlar cord, with a small lead weight attached, was lowered down the pipe Phantom grabbed the string in its 'claw' and pulled it back to the Phantom hole. In this way, ropes from each of the six support holes and from the 'electrical cable' hole were pulled back into the Phantom tent We did, of course, label them carefully.

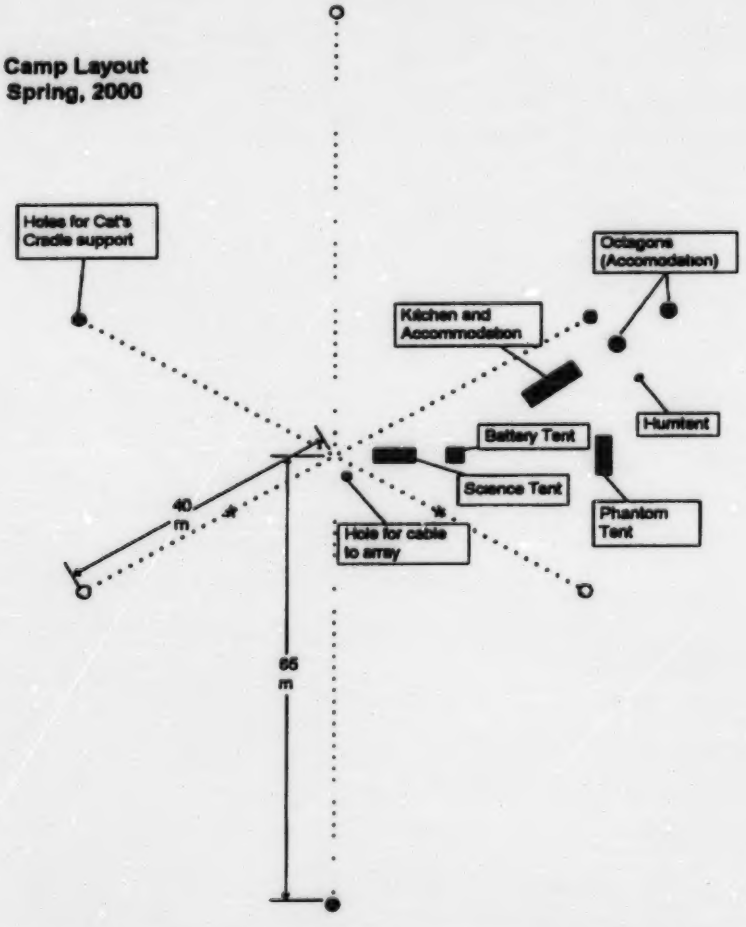

FIGURE 10. Camp layout showing the locations of the holes that were drilled for the support of the Cat's Cradle array

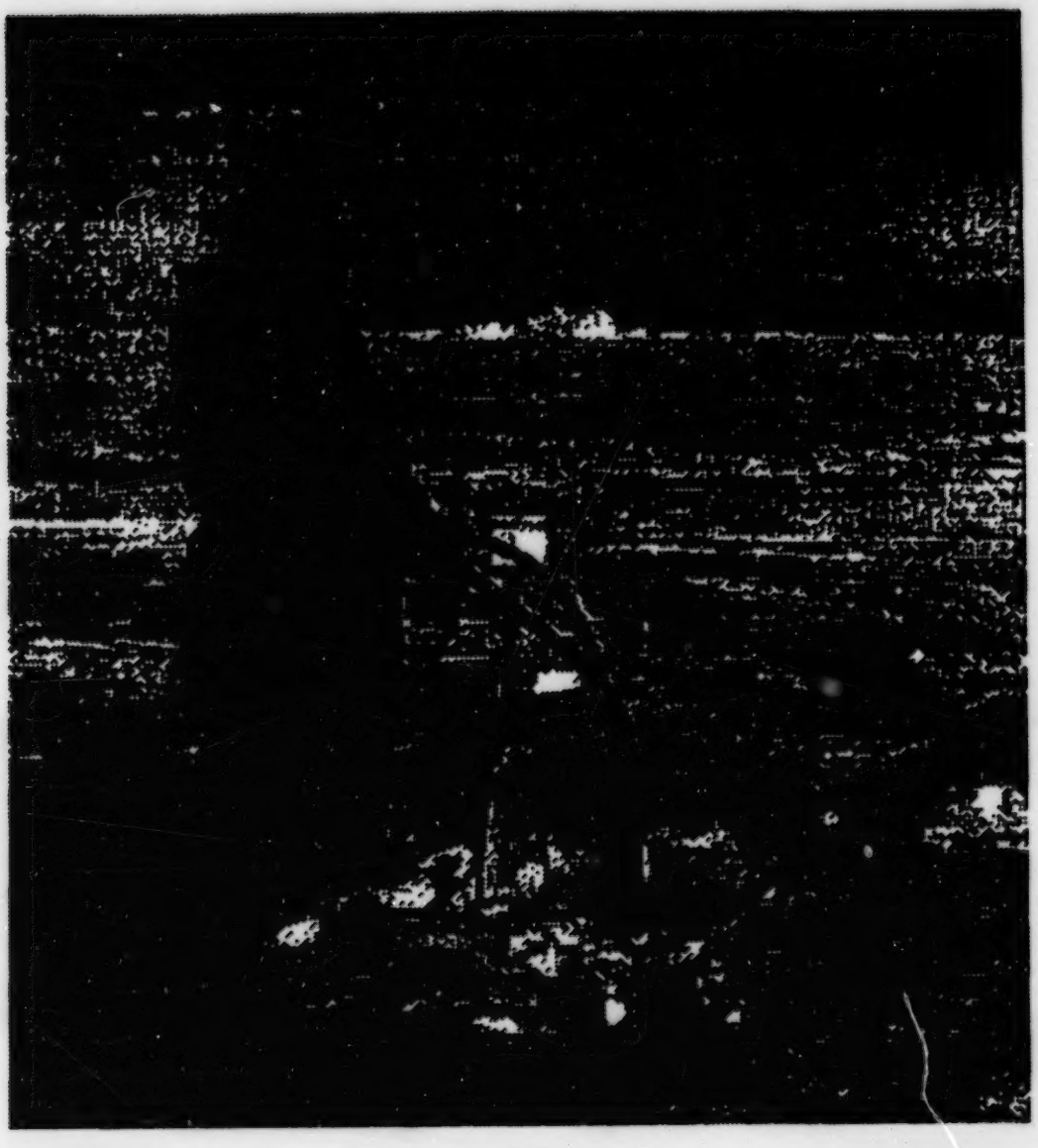

FIGURE 11 Here we are using hot water to melt out the ice in the ABS pipe This is in preparation for stringing a kevlar line, which can be seen on the spool The depression in front of the pipe is the inch hole down which we lowered a flashing light to help Phantom in its navigation

6.3 Lowering the Array

Once the ropes were all ready, we attached them to the appropriate weight (35-lb upper weight and 62-lb lower weight) and rigged the array (Figure 12), with its surgical rubber spring, between the two weights. The electrical cable, too, was tied off to its tag line When everything was ready, the weights, hydrophones, electrical cable and six support lines were lowered down the hole by yet another line, this one passing through a loop on the top weight and doubled back on itself so that it could be slipped off later. At each hole the six ropes were paid out to preset settings. When all was in place, Phantom was sent down to inspect We were pleased to find that everything was hanging freely except for one rope that was hung up at the top of a bamboo pole. (The bamboo poles were there to help keep the array straight) It took Phantom only a minute or so to lift the rope away from this snag, at which point everything looked as we thought it should.

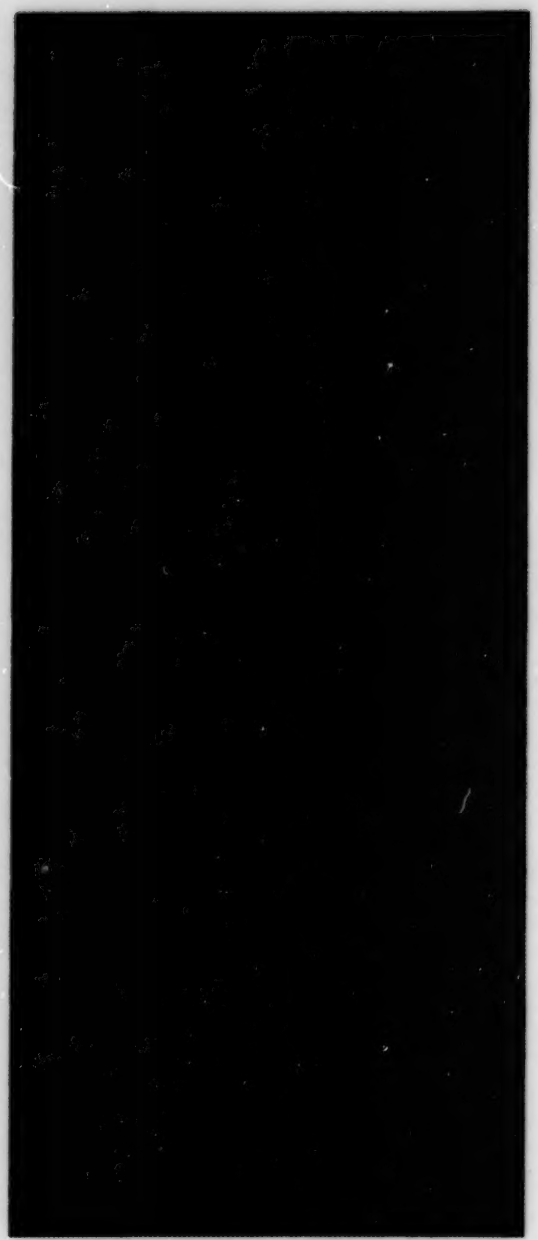

FIGURE 12 The hydrophone array laid out on the ice in preparation for being rigged to go down the ice hole

6.4 Positional Accuracy

As a check on the accuracy of this positioning technique, we performed the following experiment. We calculated the line lengths that would place the array in a vertical orientation below a spot that was not too far away from the 'origin' We then drilled a hole through the ice at this location and lowered a 10-lb weight on a light kevlar line. If our positioning was accurate, the 10-lb weight should drop directly onto the upper array weight, and its measured depth should

agree with the calculated depth of the array weight We used Phantom as our 'eye-in-the-water' to tell us when the 10-lb weight was at the right height and also to give us an estimate of the magnitude of the horizontal error

According to this measurement, the top of the array was about 25 cm to the side of the calculated position and 30 cm above it (Figure 13)

FIGURE 13 This is a picture taken from Phantom's video showing the round 10-lb ball to the right of the cylindrical 35-lb weight The ball, which is 10 cm in diameter, had been lowered from a spot directly over our calculated position of the weight As can be seen, it is about 25 cm off to the side Moreover, the ball is about 30 cm above the calculated depth of the top of the 35-lb weight

7.0 What was Learned - Problems and Possible Improvements

7.1 Geometry Could be Simplified

The 'cats cradle' of six support ropes, an electrical cable, two weights and a linear hydrophone array was very complicated to install During the installation there was a constant

fear of the ropes tangling around each other or around Phantom. At some point during the experiment we realized that all this complication was not necessary. A simpler approach would be to do away the upper weight entirely and use the ice surface as the upper support. This would have the great advantage that only one weight with its three support cables would be necessary, which would greatly reduce the probability of the ropes becoming tangled. The price that we would pay for this simplicity is that we would not be able to deploy the array horizontally; placing it at any angle near the horizontal would bring the array up close to the ice.

The deployment hole for the system should be centralized within the three support holes. This would allow the weight to be lowered without fear of tangling the ropes. Also, the weight could easily be lifted back into the hole at any time; the support lines would be long enough to reach the hole, and so they would not have to be melted out of their ABS pipes This deployment hole might be the ROV hole, but it really doesn't have to be. Once the ROV brings all the lines to this deployment hole, it is no longer needed for this job, and so its access hole might be placed elsewhere. The main disadvantage to this approach is that two large holes (rather than only one) would have to be cut through the ice. Also, the deployment hole might need to be covered by a warm tent. However, this would require extra resources, and there is a strong argument that the tent would not be necessary.

7.2 Problems with the Array

The array, itself, was too heavy and too stiff This made it hard to deploy, but, more importantly, its excess weight in water caused the surgical-rubber spring to extend to its limit. (A safety line prevented its extension past three metres.)

7.3 Surgical Rubber Hose - Alternative Materials.

The latex surgical rubber spring was a failure. After sitting for several days in the cold sea water it took a permanent set and was no longer flexible. We have learned subsequently that latex rubber becomes stiff after prolonged immersion in salt water In future, if we were to use the ice surface as the top support, a better technique for producing an (almost) constant tension would be to use a steel spring, whose properties are more reliable. This spring would be placed in the line that runs from the top of the array to the surface. The spring would have to be placed under the ice (rather than on the surface) so that the extensibility of the line would remain even after the hole froze over. The line from the spring would pass through a pipe that is frozen into the ice, and when everything was in place this

line would be pulled to a pre-determined tension with, say, a spring scale This method of determining the tension is more reassuring than is the technique of inferring it from the displacement of two objects. Also, the tension could be checked or reset later by melting the ice in the pipe and reattaching the spring scale. This would, of course, have to be done if the lower end of the cable were to be moved

7.4 Discussion of Positioning Error

The positional accuracy of 30 cm in height and 20 cm to the side is reasonably good, but we had expected errors in the vicinity of 10 cm, and we are somewhat at a loss to explain the larger error. One possible source of error was the Geodimeter. About a week after we had done the surveying, the Geodimeter failed completely. Perhaps it had been failing slowly for some time and had been giving readings that were in error

Another, more probable source of error is the kevlar cord that we used to support the weights Sometime after the experiment we discovered that woven kevlar cord has a reputation for not being perfectly stable in length To check this we remeasured the length of a couple of the lines that we had used in the deployment and discovered that they were, indeed, shorter than their original lengths by about 0.5% If we assume that this error was in effect while the cords were holding the weights in the Arctic, the new calculated location places the weight only 10 cm from its measured location

Although the positions of the weights as determined by the uncorrected line lengths were in error by some 30 cm, it should be noted that we could, in fact, obtain a very good estimate of the weight's position with the lowered measuring line that was watched by the ROV Phantom In other words, the original measurement only has to be accurate enough to steer us to the right location so that we can make the second, more accurate, set of measurements. Of course, it is to be hoped that we can improve the accuracy of the original measurement by a better choice of support lines or by a better understanding of the kevlar line.

7.5 Usefulness of a Scale Model

After performing the acoustic experiments with the array suspended vertically, we wished to repeat the sequence with the array suspended horizontally. This, of course, would mean moving the two weights so that they were at the same height and so that none of the tensions in the support cords fell to unacceptably low values. In preparing for this operation we found it very useful to build a scale model of the under-ice strings and weights. The

model showed us that if we tried to suspend the array in a horizontal orientation, some of the support cords would interfere with each other. The model indicated how far we could move the weights before the cords touched each other. We ended up tipping the array so that it was 30° to the horizontal We also discovered that, if we had positioned the ice holes for the lower support lines out farther than those for the upper lines, the two sets of lines would not have interfered with each other, and we could have tipped the array to the horizontal. However, time did not permit a restringing of the lines, and we accepted the 30° angle.

8.0 Summary and Conclusions

In a long array of hydrophones, the knowledge of the precise location of each hydrophone is very important. In the recent past, acousticians have measured these locations by acoustic techniques. However, the accuracy of these techniques could not be verified until now since there has been no way of checking the locations by an alternate method

A technique for such a check has been devised and was tried during the Arctic Field Trial of 2000 The method utilizes the fixed ice surface of the Arctic Ocean to provide a motionless and very stable platform from which to suspend the array. Once the hydrophones were rigidly fixed in a known position, several acoustic techniques were used to determine their position The analysis of these techniques will be presented elsewhere.

This report describes the six-rope support structure that held the array of hydrophones in accurately known but easily adjusted locations It derives the mathematics of positioning the array and estimating the effect of elasticity of the support ropes and the drag of the water currents. It describes a spreadsheet that was developed to use these equations to help us carry out the experiment in the field.

The spreadsheet was very useful in the design of the experiment. It helped us with the determination of the optimal hole locations, the size of the weights, the size of the suspension cords and the nature of the spring that stretches the array between the two weights. It was also very useful in allowing us to check the sensitivity of the array location to various errors in measurement. It allowed us to determine, for example, how the errors in ice-hole positions and in the lengths of the ropes will affect the position of the array.

During the spring of 2000 this suspension technique was used to suspend an acoustic array for an experiment in acoustic location. The calculated position of the two weights holding the array was checked by drilling an ice hole directly over the supposed location of the

upper weight and lowering a measuring line weighted by a 10-lb ball. The ball's location relative to the two large weights could be monitored by the Phantom ROV, and in this way the weight's location could be determined quite accurately. It was found that the calculated location was in error by about 30 cm. It should be noted, however, that the extra step of lowering the 10-lb ball on a line gave us the position of the array to an accuracy of 5 to 10 cm. This extra step, then, should be considered as part of the location procedure, and not just as a test Taken as a whole, the technique gives position accuracies of 5 to10 cm

The report discusses a number of possible improvements to the whole procedure In particular, it suggests that the probability of entanglement would be much less if only one weight were used This weight would hold the lower end of the hydrophone array, and the upper end of the array would be tensioned by a rope coming up through a hole in the ice. To make sure that this rope does not actually lift the lower weight, a steel spring would be included in its tensioning line Like the surgical rubber hose, this would ensure that small motions in the system would not change this tension by a substantial amount. The spring should preferably be installed under the ice so that when the ice hole refreezes around the rope the spring would still be effective.

It is evident that kevlar cord does not have good length stability. This must be kept in mind in any future measurement of this kind.

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A standard procedure in the underwater acoustics community is to determine the location of an array of hydrophones with acoustic techniques. Unfortunately, it is very hard to measure the accuracy of such techniques since there is nothing to which to compare the results. This report describes a way of suspending an array under the ice of the Arctic Ocean in such a way that the hydrophone locations are known accurately Weights are suspended from the ice by three lines, and the array is stretched between the weights. This system was tried out near Alert, Nunavut, Canada in the spring of 2000, and this report discusses the various strengths and weaknesses of the support system. Accuracies of about 10 cm were obtained. A number of suggestions are made for simplifying the installation.

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